## SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State Cayley – Hamilton theorem and show that it is satisfied by the matrix $A = \begin{bmatrix} -11 & -10 & 6 \\ 5 & 4 & -5 \\ -20 & -20 & 4 \end{bmatrix}$.

17. What do you mean by symmetric and antisymmetric tensors? Show that any contravariant (or) covariant tensor of second rank can be expressed as the sum of a symmetric and an antisymmetric tensor of the same rank.

18. Solve the differential equation.

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 65\cos 2x.$$

19. Prove that : $e^{\frac{x\left[z-\frac{1}{z}\right]}{2}} = \sum_{-\infty}^{+\infty} z^n J_n(x)$.

20. Derive Green's function for three dimensional Helmholtz equation.

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APRIL/MAY 2024

# GPH11/DPH11 — MATHEMATICAL PHYSICS — I

Time : Three hours Maximum : 75 marks

## SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Show that the vectors $(1, 2, -3), (1, 3, -2)$ and $(2, -1, 5)$ are linearly independent.

2. What is the characteristic equation of a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$?

3. What do you mean by real and Dummy indices in tensor analysis?

4. If $A_{ij}$ is an antisymmetric tensor, what is the value of $A_{11}$? How?

5. Solve : $(1-x^2)(1-y)\,dx = xy(1+y)\,dy$.

6. Solve the differential equation $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0$.

7. Prove that : $P_{2n+1}(0)=0$.

8. Give the Rodrigue's formula for Hermite polynomials.

9. State two properties of Delta function.

10. What is Green's function? Give its symmetry properties.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Show that the vectors $(u+v)$, $(v-v)$ and $(u-2v+w)$ are linearly independent provided $(u,v,w)$ are linearly independent.

Or

(b) Find the eigen values and normalized eigen vectors of the matrix $A=\begin{bmatrix}1&0&0\\0&0&1\\0&0&1\end{bmatrix}$.



12. (a) Prove that the addition of tensors is commutative and associative.

Or

(b) Mention the properties of Levi – Civita symbol, and show that $\sum_{jk=1}^{3}\epsilon_{ijk}\epsilon_{ljk}=2\delta_{ij}$.



13. (a) Solve the differential equation.

$$\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}.$$

Or

(b) Solve the equation

$$\frac{dy}{dx}+xy=x^3y^3.$$

14. (a) Show that $P_n(x)$ is the coefficient of $z^n$ in the expansion of $[1-2xz+z^2]^{\frac{-1}{2}}$ in ascending powers of $z$.

Or

(b) Obtain the generating function for Hermite polynomials.

15. (a) Show that $f(a,x)=\frac{1}{a}f(x)$.

Or

(b) Find one dimensional Green's function for the boundary value problem.

$y''(x)+y(x)=f(x)$, $y(0)=0$ and $y(1)=0$.